Fuzzy folding of a fuzzy group and its fuzzyretractions

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Abstract

The main target of this paper is to discuss whether the fuzzy folding of a fuzzy group is a fuzzy group or not. In addition, the relation between the fuzzy folding and the fuzzy retractions are obtained.AMS Mathematics Subject Classiftcation (2000):20D10,15H10, 57N20

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1 Introduction anddefinitions

There are many diverse applications of certain phenomena for which it is impossible to get relevant data. It may not be possible to measure essential parameters of a process such as the temperature inside molten glass or the homogeneity of a mixture inside some tanks. The required measurement scale may not exist at all, such as in the case of evaluation of offensive smells, evaluating the taste of foods or medical diagnoses by touching, see [1-10]. The aim of the current paper is to describe the above phenomena geometrically, specifically concerned with the study of the new types of fuzzy retractions and fuzzy folding of fuzzy group $(tt(\mu), \times)$. For more information about fuzzy retractions andfuzzyfoldingoffuzzygroup,onecansee[1-10].

2 Basicdefinition

To obtain our main results we will introduce the following definitions:

Deftnition 2.1. A fuzzy group $(tt(\mu), \times)$ is group which has a physical character. This character is represented by the density function μ, where $\mu \in [0, 1]$.

Deftnition 2.2. A fuzzy subgroup $(tt_1(\mu), \times)$ of a fuzzy group $(tt(\mu), \times)$ is called a fuzzy retraction of$(tt(\mu), \times)$,ifthereexistacontinuousretractionmap $\tilde{r}:(tt(\mu), \times)\to(tt_1(\mu), \times)$suchthat $\tilde{r}(a,\mu(a), \times)=(a,\mu(a), \times), \forall a\in tt_1(\mu), \mu\in[0,1]$.

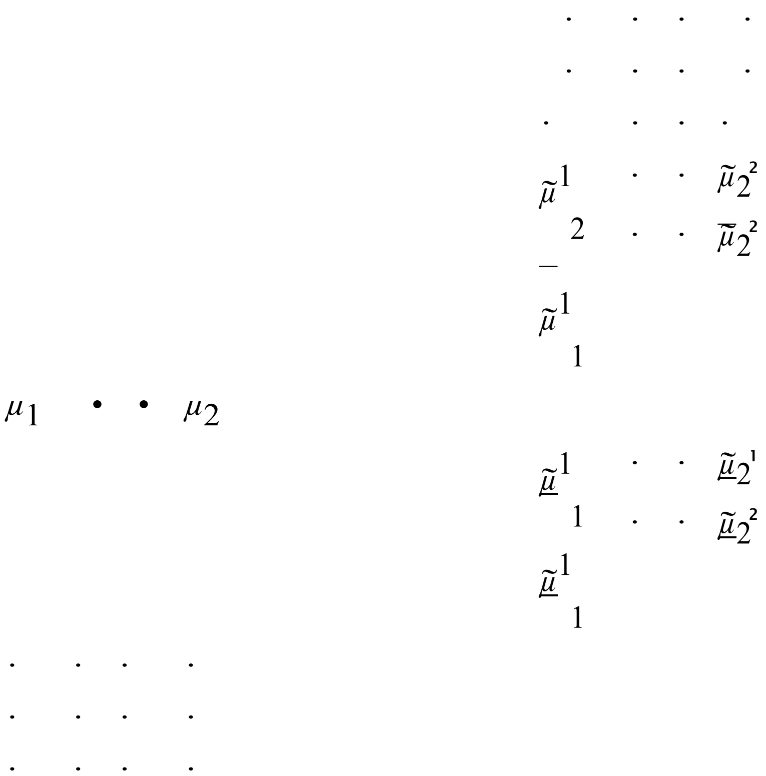

Figure 1:

: $(tt(\mu), *) \rightarrow (H(\mu), *)$,

Deftnition2.3. Thefuzzyfoldingofthefuzzygroup$(tt(\mu), \times)$isthef

uzzymap$\sim$

such that $\tilde{s}(a_i(\mu)) = (b_i(\mu)) \in H$ for all $a_i(\mu) \in tt(\mu)$. Note that, $\tilde{s}$ may not be a continuous function, and so, if $\tilde{s} : (tt(\mu), *) \to (tt(\mu), *)$, such that $\tilde{s}(tt(\mu), *) = S(\mu) \subset (tt(\mu))$, then $S(\mu)$ may not be a fuzzy group.

3 Mainresults

Corollary 3.1. *The minimum fuzzy retraction of the fuzzy group* $\{i(\mu), -i(\mu), 1(\mu), -1(\mu), *\}$ *is thefuzzysubgroup* $\{1(\mu), -1(\mu), *\}$ *whichisatwochainsofpointsupanddown* $\{1(\mu), -1(\mu)\}$. *SeeFig.(1).*

Example 1. The fuzzy rectraction of fuzzy group is a fuzzy subgroup. Let $(S_n(\mu), *)$ be a fuzzy groupofsymmetricgroupandlet $\tilde{r}$ bethefuzzyretractionmapdefinedby $\tilde{r}:(S_n(\mu),*)\to(A_n(\mu),*)$.Then$(A_n(\mu),*)$isafuzzyalternatingsubgroupoffuzzysymmetricgroup$(S_n(\mu),*)$

Example2. Thefuzzyrectractionofafuzzygroupisafuzzyset.Let$(S_n(\mu),*)$beafuzzygroup ofsymmetricgroupandlet $\tilde{r}$ bethefuzzyretractionmapdefinedby $\tilde{r}:(S_n(\mu),*)\to(\beta_n(\mu),*)$. Then $(\beta_n(\mu), *)$ is a fuzzy set of all odd permutations of a fuzzy symmetric group $(S_n(\mu),*)$.

Theorem 3.1. *The fuzzy folding of a fuzzy group is a fuzzy group if* $\tilde{s}(a) = a^{-1}$ *for all* $a \in tt(\mu)$, *and this type of a fuzzy folding represent two type of folding.*

Proof. Let $(tt(\mu), *)$ be a fuzzy group, and $\tilde{s} : (tt(\mu), *) \to (tt(\mu), *)$ such that $\tilde{s}(a) = a^{-1}$, $\forall a \in tt(\mu)$. Then $a^{-1} \in tt(\mu)$, $\forall a \in tt(\mu)$. Since $\tilde{s}(a) = a^{-1}$, $\tilde{s}(b) = b^{-1}$, so the following conditions are hold:

1. $\tilde{s}(a) * \tilde{s}(b) = a^{-1} * b^{-1}$, and since $a^{-1} * b^{-1} \in tt(\mu)$, then $\tilde{s}(a) * \tilde{s}(b) \in \tilde{s}(tt(\mu))$.
2. $\tilde{s}(a) * [\tilde{s}(b) * \tilde{s}(c)] = a^{-1} * [b^{-1} * c^{-1}] = [\tilde{s}(a) * \tilde{s}(b)] * \tilde{s}(c) = [a^{-1} * b^{-1}] * c^{-1}$, $\forall a, b, c \in tt(\mu)$.
3. $\tilde{s}(a) * q = q * \tilde{s}(a) = \tilde{s}(a) \in \tilde{s}(tt(\mu))$, and $\tilde{s}(q) = \tilde{s}(a * a^{-1}) = (a * a^{-1})^{-1} = a * a^{-1} = q \in tt(\mu)$. Therefore, $\tilde{s}(q) \in \tilde{s}(tt(\mu))$.

4. $\tilde{s}(a)$ $\tilde{s}(a^{-1})$ $\tilde{s}(a) = \tilde{s}(q) = q \in tt(\mu)$,

$= \tilde{s}(a^{-1})$

$\tilde{s}(a) * \tilde{s}(a^{-1}) = a^{-1} * a = q \in tt(\mu)$,

$\tilde{s}(a^{-1}) * \tilde{s}(a) = a * a^{-1} = q \in tt(\mu)$, and $\tilde{s}(a^{-1}) \in \tilde{s}(tt(\mu))$.

Then the fuzzy folding of a fuzzy group is a fuzzy group. Also, this type of fuzzy folding induce fuzzy upper fuzzy folding of fuzzy groups: $(\bar{tt}(\mu), *) \to (\bar{tt}(\mu), *)$ such that $\forall \bar{a} \in \bar{tt}(\mu)$, $\bar{\tilde{s}}(\bar{a}) = \bar{a}^{-1}$, and so $\bar{a}^{-1} \in \bar{t}(\mu)$, $\forall \bar{a} \in \bar{t}(\mu)$, there exists $\bar{a}^{-1} \in \bar{t}(\mu)$. Again, this type of fuzzy folding induce fuzzy lower fuzzy folding of fuzzy group $\underline{\tilde{s}} : (\underline{tt}(\mu), *) \to (\underline{tt}(\mu), *)$, such that $\forall \underline{a} \in \underline{tt}(\mu)$, there exists $\underline{\tilde{s}}(\underline{a}) = \underline{a}^{-1}$, and so $\underline{a}^{-1} \in \underline{tt}(\mu)$ $\underline{a} \in (\underline{tt}(\mu))$ $\exists \underline{a}^{-1} \in \underline{tt}(\mu)$.

□

Corollary 3.2. *If the fuzzy folding of a fuzzy group* $(tt(\mu), *)$ *is* $\tilde{s} : (tt(\mu), *) \to (tt(\mu), *)$ *such that* $\tilde{s}(a) = e$ *for all* $a \in tt(\mu)$, *then* $\tilde{s}(tt(\mu))$ *is a fuzzy group, and this type of fuzzy folding will induce upper fuzzy folding fuzzy group and lower fuzzy folding fuzzy group.*

Proof. Since $\tilde{s}(a) = q \; \forall a \in tt(\mu)$, so $\tilde{s}(a) * \tilde{s}(b) = q * q = q \in \tilde{s}(tt(\mu))$,

$\tilde{s}(a) * [\tilde{s}(b) * \tilde{s}(c)] = q * [q * q] = q \in \tilde{s}(tt(\mu))$, $[\tilde{s}(a) * \tilde{s}(b)] * \tilde{s}(c) = [q * q] * q = q \in \tilde{s}(tt(\mu))$, $\tilde{s}(q) = \tilde{s}(a * a^{-1}) = q$.

Again, this type of fuzzy folding of a fuzzy group induced fuzzy upper folding of fuzzy group, which defined by $\underline{s} : (tt(\mu), *) \rightarrow (tt(\mu), *) : \forall a \in tt(\mu), \underline{s}(a) = q$. Also, the lower fuzzy folding of fuzzy group is given by $\tilde{s} : (tt(\mu), *) \rightarrow (tt(\mu), *)$ such that $\forall a \in tt(\mu)$, there exists $\tilde{\underline{s}}(a) = q$. □

Theorem 3.2. *Let the fuzzy folding of a fuzzy group be defined by* $\tilde{s} : (tt(\mu), \circ) \rightarrow (tt(\mu), \circ)$, *such that* $\tilde{s}(a) = a$ *and* $\tilde{s}(a^{-1}) = a$. *Then the fuzzy folding of the fuzzy group is a fuzzy semigroup. Also, this type of fuzzy folding is a fuzzy retraction.*

Proof. Let $\tilde{\underline{s}} : (tt(\mu), \circ) \rightarrow (tt(\mu), \circ)$, $tt(\mu) = \{\cdots a_3^{-1}, a_2^{-1}, a_1^{-1}, e, a_1, a_2, a_3, \cdots\}$, such that $s(tt(\mu)) = \{e, a_1, a_2, a_3, \cdots\}$, and also this type of fuzzy folding is the type of fuzzy retraction defined as $\tilde{r} : (tt(\mu) \rightarrow (tt(\mu)$ such that $\bar{r}(a_i^{-1}) = a$, $\bar{r}(e) = e$, then $\tilde{s}(tt(\mu) = \tilde{r}(tt(\mu) = (S(\mu), \circ)$ is a fuzzy semi group. □

Theorem 3.3. *The limit of a fuzzy retraction of a fuzzy group is a fuzzy single element.*

Proof. Let $\tilde{r} : (tt(\mu), \circ) \rightarrow (A(\mu), \circ)$ be a fuzzy retraction map, and $(A(\mu), \circ) \subset (tt(\mu), \circ)$. Then, the a successive fuzzy retractions can be given by:

$\tilde{r}_1(A(\mu), \circ) \rightarrow (A_1(\mu), \circ)$, $\tilde{r}_2(A_1(\mu), \circ) \rightarrow (A_2(\mu), \circ)$, $\cdots$, $\tilde{r}_n(A_{n-1}(\mu), \circ) \rightarrow (A_n(\mu), \circ)$, where $A_n(\mu) \subset A_{n-1}(\mu) \subset \cdots \subset A_1(\mu)$ and $\lim_{n \to \infty} \tilde{r}_n(A_n(\mu), \circ) = \{a\}$. Again, this type of fuzzy retraction induce upper and lower fuzzy single points, $\bar{a}$ and $\underline{a}$. □

Corollary 3.3. *The limit of a fuzzy folding of a fuzzy group is a fuzzy single point.*

Proof. Let $\tilde{s}_1 : (tt(\mu), \circ) \rightarrow (tt(\mu), \circ)$ such that $\tilde{s}_1(tt(\mu)) = tt_1(\mu) \subset tt(\mu)$, $\tilde{s}_2(tt(\mu)) = tt_2(\mu) \subset tt(\mu), \cdots, \tilde{s}_n(tt_{n-1}(\mu)) = tt_n(\mu)$, and $\lim_{n \to \infty} s_n(tt_n(\mu)) = \{a\}$. Also, this type of fuzzy folding induce upper and lower fuzzy single points $\bar{a}$ and $\underline{a}$. □

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